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Title of Proposal

Seismic reliability of railway bridges under high-speed trains using a novel formulation for constrained dynamical systems

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CONTENTS

1 BRIDGE & HIGH-SPEED TRAIN INTERACTION: STATE-OF-THE-ART REVIEW

1.1 State of the art

The investment planning and decision-making for an upgrade and expansion of existing roadway and railway networks are based on detailed technical studies accounting for various social and economic parameters. The decision-making procedure includes assessment of critical infrastructure (e.g., bridges) and evaluation of development potentials. In this context, a robust and reliable analysis of the coupled train-railway-bridge system can provide a valuable tool for assessment of the existing network considering the effects of high-speed train and bridge interaction during dynamic (i.e., seismic) loading.

The dynamic behavior of the coupled system vehicle (train) – track - bridge system is strongly coupled and complex. In general, the vehicle subsystem can be described as a multibody assembly, and the track - bridge subsystem can be modeled using classical structural finite element formulations. The subsystems interact through contact/impact forces, i.e., forces between the vehicle wheels and the rail on the bridge deck. With the rapid development of high-speed railways, the vehicle(train)–track–bridge (VTBI) interaction draws growing attention of scholars worldwide in recent decades [1]. Numerous VTBI models were proposed to investigate the dynamic performance of the coupled system, which can be organized into the following categories according to the adopted simplifications.

The moving force model (MFM) is the simplest and earliest model, where train subsystem was modeled as constant forces. Utilizing this model, Kolousek [2] simulated the unbalanced forces of steam locomotives as single moving forces to consider the vertical vibration of continuous girder bridges. The moving force model assumes that the inertia of the moving train is small compared to the inertia of the bridge. However, the overall accuracy is poor due to the oversimplification of the vehicle subsystem.

Subsequently, the moving harmonic force model (MHFM) was proposed in the early twentieth century, where the eccentric forces of locomotives were considered as moving harmonic forces, e.g. Krylov[3] and Timoshenko [4] investigated the resonance of bridge structures. However, the dynamic interaction between vehicle and bridge was also not considered in this model. This formulation yields acceptable results if the dynamic behavior of the train is not of interest.

Moreover, if the dynamic effect of the vehicle cannot be ignored, the moving mass model (MMM) should be utilized, which considers the mass and inertia of the running vehicle. This model was first proposed and solved by Willis [5] and Stokes [6]. Important improvements were proposed by Jeffcott [7], Inglis [8]. Since the suspension systems of the vehicle are not included, the moving mass model cannot accurately simulate the dynamic effects of moving trains on bridges.

Furthermore, the moving sprung-mass model (MSMM) was developed by considering the effect of the suspension system of the train, leading to a model of a moving mass supported by a spring-damping element (Biggs and Testa [9]).

After that, the description of the vehicle as 4 degrees of freedom rigid-body (two-axle vehicle model (TAVM)) was introduced, which is more like the modern vehicle-bridge interaction model. In this case, also the vertical and pitch motions of the sprung part were considered [10,11].

Later, the theory of multi-body dynamics (MBD) is adopted to simulate vehicle (train) subsystem, while the bridge subsystem is described via FEM or continuous beam theories. Relevant work conducted by Chu et al. [12], Bhatti [13], Olsson [14], Diana and Cheli [15], Green and Cebon [16], Yang and Lin [17], Xia et al. [18]. Moving vehicle system models have been used for different purposes [9–18, 19, 20, 21, 22, 23].

In general, the vehicle is represented as MBD system (rigid or flexible). A comprehensive literature review of railway vehicle MBD modeling is provided in [24–26]. In general, the DOFs of the vehicle model range according to the main issue of interest [11, 21, 27-29]]. The vertical VTBI usually include DOFs that are related to the vertical and pitch motions of the vehicle [19, 30]. In the model proposed by Vu-Quoc and Olsson [31], the longitudinal DOFs of the vehicle components were also included to simulate the speed variation of the vehicle. Also, full 3D MBD model are widely applied [19, 21, 27-29]. These models fully consider the 3D coupling behavior between vehicle components through bogie suspensions. This model can handle nonlinear stiffness and damping properties of the primary and the secondary suspensions in three directions, e.g. nonlinear yaw dampers and secondary lateral bump-stop clearances, etc. [32].

In general, five DOFs are taken into consideration for each rigid body, describing vertical, lateral, roll, yaw and pitch motions. These models can simulate the 3D dynamic response of railway vehicles moving on bridges, as well as evaluating the ride comfort and running safety of the train. This model can, of course, be further extended to the 3D TTBDIM by considering the longitudinal DOFs of the vehicle components. A vehicle modeled as a rigid body with six DOFs presented by Ling et al. in paper [33]. As noted, there are some essential nonlinearities in the railway vehicle dynamics system such as wheel-rail contact and suspension components. The accuracy of multi-body models is mainly affected by the representation of wheel-rail contact and by the modeling of suspension components [34,35]. These modeling assumptions are more important in case of high-speed trains equipped with hydraulic dampers, friction elements, nonlinear lateral bump-stop, Coulomb friction and clearances between the axle box and the side frame, etc. [28, 29, 36, 37]. Another issue is the possible significant effects of structural flexibility. In general, the flexibility of car body affects the dynamic response and ride comfort of railway vehicles. These effects investigated in [38–41] utilizing a simplified model based on Euler beams supported by bogie suspensions. The main conclusion is that flexibility affects mainly vehicle response and the ride comfort, while they're no practical influence on the bridge response.

The basic limitation of the traditional VTBI model was gradually realized by engineers, that is the influence of track vibrations were not considered. The vehicle, the track and the bridge necessarily form an integrated dynamic system, in which the vehicle and the track are coupled by the wheel-rail interactive relationship, and the track and the bridge are connected through the track–bridge interaction. Different track structures lead to modification of the wheel-rail forces, that finally influence the vibration of bridges through the track–bridge interaction and vice versa. Therefore, including a detailed track model into the VTBI is critical, Zhai [19-21, 29]. Also, Cheng et al. proposed a type of vehicle–track–bridge element where the vehicle is modeled as a sprung-mass system, while the rail and the bridge deck are all modeled as beam elements [43]. Furthermore, Wu and Yang proposed a vehicle–rail–bridge interaction model and some types of vehicle–rail interaction elements based on dynamic condensation method [44], whereas Biondi et al. [45] applied a substructure approach which was used to investigate the dynamic responses of multi-span simply supported bridges. Moreover, Lou [46] proposed a model, where the vehicle was modeled as a sprung mass system while the rail and the bridge were simulated as Bernoulli beam elements To control the vibrations in the train–track–bridge system. Also, Martínez-Rodrigo et all. [47] studied the transverse vibrations of railway bridges under resonant conditions. The influence of ballasted track structure investigated by Rigueiro et al. [48]. Also, a 3D rail–ballast–beam finite element model was proposed by Guo et al. [49]. In a work of Yang and Hwang [50], a methodology for train-track-bridge interaction was proposed based on the direct stiffness method and the mode superposition method. Zhu et al. [51] proposed a hybrid solution to solve vehicle-track-bridge interaction. To consider the vibrations of track structures, the traditional train–bridge interaction model has been extended to the VTBI interaction model.

Although the ballasted track is widely used, rail bridges can also contain non-ballasted tracks. The railway track is a key structure that distributes the load on bridge structure. Therefore, rail bridges can contain either the ballasted track or the non-ballasted tracks. The ballasted track system consists of rail, railpad, fastening, sleeper, and ballast. The type 'Double-block' track consists of rail, railpad, fastening, concrete sleeper, concrete slab, and concrete base. The rails are fastened onto sleepers in two blocks, which are directly concreted into a cast-in-place slab. In this case only the vibration of rails is important for vehicle–track interaction. The type 'Elastic-supporting-block' track consists of rail, railpad, fastening, concrete block, block pad, rubber boot, and concrete base. The flexibility induced by the rubber boot render vibrations of rails and concrete blocks important for the vehicle–track. Slab tracks are widely used in high-speed railways, which consists of rail, railpad, fastening, concrete slab, CAM layer, and concrete base. The elastic bearings separate the track from the bridge deck, yielding a floating slab track. Both the vibrations of the rails and slabs significantly affect the vehicle–track interaction. It is well known that track dynamic behavior affects train running safety and ride comfort [24-26, 52]. Therefore, a detailed track model is necessary for the accurate analysis of the VTBI systems. A survey of track modeling methods for vehicle–track interaction can be found in [24-26]. There are three basic categories of track simulation : (a) lumped parameter models, which includes rigid mass or massless elements for rails supported on sleepers or on sleepers and ballasts connected by spring and damping elements [24-26]. (b) Continuous beam models, where the rails are usually represented by infinite Euler or Timoshenko beams resting on Winkler elastic foundations [24-26] or discrete sleepers along the track [20, 21, 24-26] and (iii) discrete beam models, where finite element (FE) method is applied [24-26, 45,52-55]. In FE modelling, tracks are modeled as mass-spring–dashpot models [48, 49, 52, 56] or solid models [52, 57]. In continuous models, the rails are usually represented by infinite Euler, or Timoshenko beams resting on Winkler elastic foundations [24–26] or discrete sleepers along the track [21, 24-26, 29]. Discrete models usually include finite rails modeled using the finite element (FE) method [24-26, 45, 52, 53-55]. In FE modelling, two classes of track models can be distinguished: mass-spring–dashpot models [48, 49, 52, 56] and solid models [52]. The continuous and discrete beam models are more accurate and take into account the rails, the fastenings, the sleepers, the slabs, and even the ballast [52, 58].

The number of layers of the track models in VTBI can vary from one up to four. A single layer model for the case of a double-block non-ballasted track shown to be adequate. In case of the two-layer track models [48, 56], the concrete sleepers or blocks are usually represented by rigid bodies, while the elastic bearing layers are modelled as linear or nonlinear viscoelastic elements. The track slabs can be modeled as beams in 2D models, whereas in 3D models they are simulated as elastic plates supported on the viscoelastic foundation [29, 32]. Usually, the three-layer and four-layer track models are developed for the ballasted track system, which contains rails, sleepers, ballast, or subgrade [29, 52].

Comparison on the ballasted track models with different layers presented [43, ,45, 48]. In general, the track model influences the vehicle response and the wheel-rail contact forces.Bridge structure is mainly simulated either by analytical methods either by the finite element method (FEM). Research on the vibration of bridges by using analytical or semi-analytical approaches is abundant. In case of analytical or semi-analytical methods bridges are modeled base on beam theories, [2, 3, 4, 21], where Timoshenko [4] obtained approximate solutions to the problem of simple beams under moving loads. Also, Vellozzi [59] and Zha et al. [19] developed some semi-analytical bridge model for the case of vertical vehicle–track–bridge interaction. Simulation-based on FEM, BEM (boundary element method) and the F–B hybrid method has been developed [21, 28, 52]. Nowadays, FEM is extensively used to formulate the bridge models in VTBI. According to the type of bridge structures, beam element, plate/shell element, and 3D solid elements can be used for modeling of the various components. Simply supported bridges can be easily modeled by 3D beam elements [19, 28, 29], while the FE models for the more complicated bridges usually consist of a combination of beam elements, shell and solid elements [19, 21, 52, 60]. Direct integration methodologies are adopted to solve the spatially discretized equations of the bridge FE models for the case of a general bridge [52]. Some approximation assumptions such as the modal superposition method, or component mode synthesis techniques have been adopted in the FE [28], retaining a preselected range of frequencies, that lead to a significant reduction of computational cost. But it should be mentioned that the selection of the cut-off frequency depends on the main scope of the analysis, it is different in the main interest is ride comfort or the internal forces on the beam structure [52].

The contact forces between the wheels and the rails realize the coupling of the vehicle subsystem with the track subsystem at the wheel-rail interfaces [61]. The two most common methods to compute the contact forces are the kinematic constraint and the compliance method [62]. The kinematic constraint method assumes no penetration occurs between the wheels and the rails [21, 62-65]. Thus, the normal contact force is derived from the imposition of an impenetrability constraint. In the compliance method, the normal contact force is a function of the stiffness (and damping) and the penetration between contacting bodies. This function can be either linear or nonlinear (e.g., the Hertz's contact theory) [62]. Also, the assumption that the displacement of a wheel is always equal to that of the bridge/rail has been often adopted in the conventional train–bridge dynamic interaction analyses [29, 44, 66]. In order to take into account wheel/rail geometric contact some approaches were presented in [29, 57, 67, 68]. These models deal with the loss of contact between the wheel and the rail, whereas curved and worn wheel/rail profiles are included as well.

The wheel-rail contact forces include the normal contact force and the tangent creep forces. The nonlinear Hertzian elastic contact theory is usually used to calculate the wheel-rail normal contact forces [29, 62]. The normal forces could also be obtained by solving a linear complementary problem (LCP) [69, 70], utilizing, for example, Lemke complementary pivot algorithm [70]. During rolling contact, the leading contact area is in sticking, while the remaining area might slip. This small apparent slip is called creep [71]. Creep depends on the relative velocities of at the contact point and is crucial for the determination of the tangential forces, as well as, for the steering and the stability during earthquakes [62].

For the study of train running stability and safety under crosswinds or earthquakes, advanced contact theories like the Kalker linear or nonlinear creep theories should be applied. For the calculation of wheel-rail nonlinear creep forces, the Kalker's 3D exact theory (CONTACT) is used, whereas some recent works show that the 3D finite element method (FEM) could also solve the 3D wheel/rail contact problem with acceptable accuracy in comparison to the CONTACT [72–74]. To reduce the computational cost some approximate models, such as the FASTSIM, USETAB, Vermeulen–Johnson, Shen–Hedrick–Elkins, Polach methods, are commonly used for vehicle–track–bridge dynamic simulations [29, 57, 67-68, 75].

It is noted that the linear creep theory by Kalker is valid only for small creepage, where no slip occurs at the interface, and the creepages are in the region of adhesion. But when the. The nonlinear models such as the Shen–Hedrick–Elkins model [76] or the Kalker FASTSIM algorithm [87] should be used for the calculation of the creep forces in case of large creepages.

For the bridges where ballasted tracks are used, the loads are transferred to the bridge deck through the ballast layers. While for the non-ballasted tracks, an elastic bearing layer (such as CAM or rubber pads) is often utilized to connect the track slabs and the bridge decks. In either case, ballasted or non-ballasted, the track–bridge interaction can be discretized into point-to-point connections, which are connected with linear spring and dampers at each contact point.

Track irregularities are the most important excitation source. The simulation of irregularities follows the spectral representation method, described by power spectral density (PSD) functions of the track geometry quality. The commonly used PSD functions are the Federal Railroad Administration (FRA) spectrum [21], the German track spectrum [11, 21], the SNCF track spectrum [11, 52], the Chinese track spectrum [29] and PSD functions proposed by ISO 3095 [77]. For the case of time-domain analyses, irregularities are obtained by performing inverse Fourier transforms with random phase see Dinh et al. [67], Schiehlen [78].

Also, various geometric defects at wheel-rail interfaces are significant excitation source for the fully coupled system. These include rail corrugations, the out-of-round wheels or the rounded flat wheels, the defect of welded-rail and dipped-rail joints, etc. Usually this type of defects is described by harmonic waves . The periodic geometry defects are treated as an addition to the elastic compression at the wheel-rail contact zones. On the other hand, the localized geometry defects are represented by cosine displacements [33] or the impact velocities .

Other sources of excitation that affect the safety of the coupled system are earthquake, crosswind, and collisions. The dynamic response of the vehicle–track–bridge system subjected to crosswinds is investigated in [21, 28, 79-81]. As input, the artificial wind velocity time series is adopted [28]. A stochastic wind velocity field is utilized through the spectral representation method, linear filtering method, the wavelet method and the artificial wind field based on observed records [28]. Note that it is essential to reproduce realistic wind velocity time histories describing at different locations along the span and different heights.

The operational safety of HSR trains over bridges during earthquakes is an issue of great research interest [63]. In most cases, derailment is estimated indirectly through either force-based metrics [36, 63, 82, 83] or geometric criteria [84-86]. The so-called derailment factor of a single wheel is equal to the lateral contact force L over the vertical one V (L/V ratio) and dates back more than one hundred years. This method is a conservative prediction of the derailment [87] and is based on the behavior of a single wheel [82]. Weinstock [88] proposed a derailment criterion by summing the absolute L/V values of the two wheels of the same wheelset. These criteria do not cover the case of wheel-rail-flange contact and wheel climbing [86].

Based on a simplified model, Luo in [89] provided insight into the mechanism of the derailment and identified the areas where rocking or swaying derailment prevails. Also, Tanabe et al. in [85] assumed that the derailment happens during an earthquake when the relative displacement between the wheel and the rail in the lateral direction exceeds geometric derailment criteria. Furthermore, Du et al. [90] described the problem of detachment based on a Hertz contact model and showed that the probability and the duration of derailment increase with the train speed. Also, Ju [91] proposed a contact element that simulates sticking, sliding and wheel-rail separation and studied the behavior of a single wheel. The numerical method should be able to describe directly different wheel-rail contact states of the coupled vehicle-track–bridge systems such as flange contact, wheel climbing-up, wheel-rail detachment, and derailment. Also, it should cover practical nonlinear profiles of the wheels and the rails.

When the dynamic response of a train–track–bridge system subjected to an earthquake is studied, the time histories of the seismic motions around the bridge piers are essential. The seismic ground motion for the analysis of TTBDI can be obtained from strong seismic motion records [18,21, 92, 93], which are usually rare and insufficient. As a result, the application of artificial ground motions generated by numerical simulations is a good option. The seismic ground motion is a typical non-stationary process, and the motion samples can be obtained by using the random field theory based on spectral method [28,93]. Two numerical methods, namely the unconditional and conditional simulations have been widely used to obtain the random field of seismic ground motion [28].

In the case of a long bridge, the effects of the spatial variation of the seismic ground motion, asynchronous excitation should be considered.

The coupled vehicle–track–bridge dynamic interaction system contains strong nonlinearities arising from the wheel-rail contact, suspension elements, bridge-structure and in general large number of dofs. Therefore, it is crucial to develop a high-efficiency numerical algorithm to solve such a large-scale system. Mainly, numerical schemes in the time domain are applied [21, 28, 52, 94-97]. However, some frequency domain methods with high computational efficiency have also been presented by Green and Cebon [16].

The solution of the coupled system in time domain utilizes a time integration technique like the Newmark method [95], the HHT-α method [96], the Wilson method [97] and the Zhai method [94] is applied. Many techniques are tailored for the specific problem, that decomposes the coupled system to two or more subsystems to maximize computational efficiency. Usually explicit time integration schemes are applied for the vehicle and track subsystems whereas, the bridge FE models are usually solved by employing an implicit integration scheme. Such a hybrid explicit–implicit integration scheme was proposed by Zhai et al. [29] for the vehicle -track subsystem a simple fast explicit two-step integration method namely ‘Zhai method’ was adopted, while the Newmark-β implicit integration method was utilized on analysis of the bridge structure dynamics.

Summarizing time-integration algorithms developed for the VTBI problem, either directly solve the coupled vehicle-track-bridge system (coupled algorithms) [17, 21, 63-66], or solve the vehicle subsystem and bridge subsystem separately in a co-simulation environment (iterative algorithms) [28]. A primary characteristic of the coupled algorithms is that they lead to time-depended system quantities [21, 63–65] due to the presence of moving (wheel-rail) contact forces that act as loads within the coupled vehicle-bridge system. Therefore, the global system matrices must be updated and factorized at each time step, leading to a considerable computational effort [98]. Besides, since coupled algorithms solve the equations of motion (EOMs) of the vehicle and the bridge subsystems together, a standard numerical algorithm is used, whereas the subsystems possess different dynamical characteristics.

Regarding iterative methods, they solve each subsystem independently, and this procedure terminates when a convergence criterion is fulfilled, e.g. a compatibility condition is within a predefined tolerance. Xia et al. [99] proposed such a scheme that treats the contact forces as external loads to the vehicle and the bridge subsystems respectively and solves the two subsystems separately. Liu et al. [100] referred to this algorithm as a loosely coupled iterative algorithm and applied it to analyze a composite bridge under HSR trains. More recently, Zhang and Xia [101] proposed a refined iterative algorithm, in which the first trial of the vehicle response is predicted by setting the bridge motion to zero.

Neves et al. [98] proposed a methodology that also avoids the use of time-dependent system matrices. Instead, it solves the system of linear equations and the compatibility condition between the two sub-systems simultaneous.

An alternative approach that potentially incorporates the advantages of both coupled [63-65] and iterative schemes [99,100] is the localized version of the Lagrange multipliers method (Park et al. [102, 103]). This approach introduces an additional artificial auxiliary point at the contact interface, it assigns two sets of Lagrange multipliers and states two sets of kinematic constraints between the extra point and the other contact points.

References

[1] Xia H, Zhang N. Train–track–bridge dynamic interaction: a state-of-the-art review. Vehicle System Dynamics 2019;7:984-1027.

[2] Koloušek V, McLean RF, Fleming JF. Dynamics in engineering structures. London: Butterworths. 1973.

[3] Krylov AN. Mathematical collection of papers of the Academy of Sciences. Vol. 61. St.

Petersburg. Russia. 1905.

[4] Timoshenko SP. On the forced vibrations of bridges. London Edinburgh Dublin Philos Mag J Sci. 1922;43(43):1018–1019.

[5] Willis R. Preliminary essay to the Appendix B: Experiments for determining the effects produced by causing weights to travel over bars with different velocities. Report of the com- missions appointed to inquire into the application of iron to railway structures, London: W. Clowes and Sons. 1849.

[6] Stokes SGG. Discussion of a differential equation relating to the breaking of railway bridges.

Printed at the Pitt Press by John W. Parker. 1849;8:707–735.

[7] Jeffcott HH. On the vibration of beams under the action of moving loads. London Edinburgh Dublin Philos Mag J Sci. 1929;8:66–97.

[8] Inglis CE. A mathematical treatise on vibrations in railway bridges. Cambridge: Cambridge University Press; 2015.

[9] Biggs JM, Testa B. Introduction to structural dynamics. New York (NY): McGraw-Hill; 1964.

[10] Wen RK. Dynamic response of beams traversed by two-axle loads. J Eng Mech Div.1960;86:91–112.

[11] Frýba L. Vibration of solids and structures under moving loads. Springer Science& Business Media; 1999.

[12] Chu KH, Dhar CL, Garg VK. Railway-bridge impact: simplified train and bridge model. J Struct Div. 1979;105:1823–1844.

[13] Bhatti MH. Vertical and lateral dynamic response of railway bridges due to nonlinear vehicles and track irregularities [dissertation]. Chicago: Illinois Institute of Technology; 1982.

[14] Olsson M. Finite element, modal co-ordinate analysis of structures subjected to moving loads.

J Sound Vib. 1985;99:1–12.

[15] Diana G, Cheli F. Dynamic interaction of railway systems with large bridges. Veh Syst Dyn.

1989;18:71–106.

[16] Green MF, Cebon D. Dynamic response of highway bridges to heavy vehicle loads: theory and experimental validation. J Sound Vib. 1994;170:51–78.

[17] Yang YB, Lin BH. Vehicle-bridge interaction analysis by dynamic condensation method. J Struct Eng. 1995;121:1636–1643.

[18] Xia H, Han Y, Zhang N, et al. Dynamic analysis of train–bridge system subjected to non-uniform seismic excitations. Earthq Eng Struct Dyn. 2006;35:1563–1579.

[19] Zhai WM, Cai CB. Train/track/bridge dynamic interactions: simulation and applications. Veh Syst Dyn. 2002;37:653–665.

[20] Zhai WM, Cai CB, Wang KY, et al. Numerical simulation and field experiment of high-speed train-track-bridge system dynamics. Veh Syst Dyn. 2004;41:677–686.

[21] Yang YB, Yau JD, Wu YS. Vehicle–bridge interaction dynamics. Singapore: World Scientific Publishing Co. Pte. Ltd.; 2004.

[22] Bhatti MH, Garg VK, Chu KH. Dynamic interaction between freight train and steel bridge. J Dyn Syst Meas Control. 1985;107:60–66.

[23] Cantero D, Arvidsson T, OBrien E, et al. Train–track–bridge modelling and review of parameters. Struct. Infrastruct. Eng. 2016;12:1051–1064.

[24] Iwnicki S. (Ed.). Handbook of railway vehicle dynamics. Boca Raton: CRC/Taylor & Francis;

2006.

[25] Knothe KL, Grassie SL. Modelling of railway track and vehicle/track interaction at high

f requencies. Veh Syst Dyn. 1993;22(3–4):209–262.

[26] Popp K, Kruse H, Kaiser I. Vehicle-track dynamics in the mid-frequency range. Veh Syst Dyn.

1999;31:423–464.

[27] Arvidsson T, Andersson A, Karoumi R. Train running safety on non-ballasted bridges. Int J Rail Transp. 2019;7(1):1–22.

[28] Xia H, Zhang N, Guo W. Dynamic interaction of train-bridge systems in high-speed railways: theory and applications. Berlin: Springer; 2018.

[29] Zhai WM, Xia H, Cai C, et al. High-speed train–track–bridge dynamic interactions – part I: theoretical model and numerical simulation. Int J Rail Transp. 2013;1:3–24.

[30] Neves SGM, Montenegro PA, Azevedo AFM, et al. A direct method for analyzing the nonlinear vehicle-structure interaction. Eng Struct. 2014;69:83–89.

[31] Vu-Quoc L, Olsson M. A computational procedure for interaction of high-speed vehicles on flexible structures without assuming known vehicle nominal motion. Comput Methods Appl Mech Eng. 1989;76:207–244.

[32] Zhai WM, Wang KY, Cai CB. Fundamentals of vehicle–track coupled dynamics. Veh Syst Dyn.

2009;47:1349–1376.

[33] Ling L, Dhanasekar M, Thambiratnam DP. Dynamic response of the train–track–bridge system subjected to derailment impacts. Veh Syst Dyn. 2018;56:638–657.

[34] Bruni S, Vinolas J, Berg M, et al. Modelling of suspension components in a rail vehicle dynamics context. Veh Syst Dyn. 2011;49:1021–1072.

[35] Evans J, Berg M. Challenges in simulation of rail vehicle dynamics. Veh Syst Dyn.
2009;47:1023–1048.

[36] Ju SH. Nonlinear analysis of high-speed trains moving on bridges during earthquakes.
Nonlinear Dyn. 2012;69:173–183.

[37] Tanabe M, Sogabe M, Wakui H, et al. Exact time integration for dynamic interaction of high-speed train and railway structure including derailment during an earthquake. J Comput Nonlinear Dyn. 2016;11(3):031004. doi:10.1115/1.4030829.

[38] Wang K, Xia H, Xu M, et al. Dynamic analysis of train-bridge interaction system with flexible car-body. J. Mech. Sci. Technol. 2015;29:3571–3580.

[39] Li Q, Xu YL, Wu DJ, et al. Computer-aided nonlinear vehicle-bridge interaction analysis. J Vib Control. 2010;16:1791–1816.

[40] Younesian D, Marjani SR, Esmailzadeh E. Importance of flexural mode shapes in dynamic analysis of high-speed trains traveling on bridges. J Vib Control. 2014;20:1565–1583.

[41] Ling L, Zhang Q, Xiao X, et al. Integration of car-body flexibility into train–track coupling
system dynamics analysis. Veh Syst Dyn. 2018;56:485–505.

[42] Zhai WM, Sun X. A detailed model for investigating vertical interaction between railway vehicle and track. Veh Syst Dyn. 1994;23:603–615.

[43] Cheng YS, Au FTK, Cheung YK. Vibration of railway bridges under a moving train by using
bridge-track-vehicle element. Eng Struct. 2001;23:1597–1606.

[44] Wu YS, Yang YB, Yau JD. Three-dimensional analysis of train-rail-bridge interaction prob- lems. Veh Syst Dyn. 2001;36:1–35.

[45] Biondi B, Muscolino G, Sofi A. A substructure approach for the dynamic analysis of train- track-bridge system. Comput Struct. 2005;83:2271–2281.

[46] Lou P. Finite element analysis for train-track-bridge interaction system. Arch Appl Mech. 2007;77:707–728.

[47] Martínez-Rodrigo MD, Lavado J, Museros P. Transverse vibrations in existing railway bridges under resonant conditions: single-track versus double-track configurations. Eng Struct. 2010;32:1861–1875.

[48] Rigueiro C, Rebelo C, da Silva L S. Influence of ballast models in the dynamic response of railway viaducts. J Sound Vib. 2010;329:3030–3040.

[49] Guo WW, Xia H, De Roeck G, et al. Integral model for train-track-bridge interaction on the Sesia viaduct: dynamic simulation and critical assessment. Comput Struct. 2012;112:205–216.

[50] Yang SC, Hwang SH. Train-track-bridge interaction by coupling direct stiffness method and mode superposition method. J Bridge Eng. 2016;21:1–16.

[51] Zhu Z, Gong W, Wang L, et al. A hybrid solution for studying vibrations of coupled train–track–bridge system. Adv Struct Eng. 2017;20:1699–1711.

[52] Arvidsson T, Karoumi R. Train–bridge interaction – a review and discussion of key model parameters. Int J Rail Transp. 2014;2:147–186.

[53] Rocha JM, Henriques AA, Calçada R. Safety assessment of a short span railway bridge for high-speed traffic using simulation techniques. Eng Struct. 2012;40:141–154.

[54] Zhu S, Cai C, Zhai W. Interface damage assessment of railway slab track based on reli- ability techniques and vehicle-track interactions. J Transp Eng. 2016;142(10):04016041. doi:10.1061/(ASCE)TE.1943-5436.0000871.

[55] Zhu S, Wang J, Cai C, et al. Development of a vibration attenuation track at low frequencies

for urban rail transit. Comput-Aided Civ Inf. 2017;32:713–726.

[56] Neves SGM, Azevedo AFM, Calçada R. A direct method for analyzing the vertical vehicle–structure interaction. Eng Struct. 2012;34:414–420.

[57] Li Q, Xu YL, Wu DJ. Concrete bridge-borne low-frequency noise simulation based on train – track – bridge dynamic interaction. J Sound Vib. 2012;331:2457–2470.

[58] Zhu S, Cai C, Spanos PD. A nonlinear and fractional derivative viscoelastic model for rail pads in the dynamic analysis of coupled vehicle–slab track systems. J Sound Vib. 2015;335:304–320.

[59] Vellozzi J. Vibration of suspension bridges under moving loads. J Struct Div. 1967;93:123–138.

[60] Cai C, He Q, Zhu S, et al. Dynamic interaction of suspension-type monorail vehicle and bridge: numerical simulation and experiment. Mech Syst Signal Process. 2019;118:388–407.

[61] Q. Zeng, and E.G. Dimitrakopoulos. Vehicle-bridge interaction analysis modeling derailment during earthquakes. Nonlinear Dynamics. 2018; 93-4, 2315-2237.

[62] Shabana, A.A., Zaazaa, K.E., Sugiyama, H.. Railroad Vehicle Dynamics: A Computational Approach. CRC Press: New York; 2010.

[63] Q. Zeng, and E.G. Dimitrakopoulos. Seismic response analysis of an interacting curved bridge-train system under frequent earthquakes. Earthq. Eng. Struct. Dyn. 2016; 45, 1129-1148.

[64] Zeng, Q., Yang, Y.B., Dimitrakopoulos, E.G. Dynamic response of high speed vehicles and sustaining curved bridges under conditions of resonance. Eng. Struct. 2016; 114, 61-74.

[65] Dimitrakopoulos, E.G., Zeng, Q.: A three-dimensional dynamic analysis scheme for the interaction between trains and curved railway bridges. Comput. Struct. 2015; 149, 43-60.

[66] Xia H, Zhang N. Dynamic analysis of railway bridge under high-speed trains. Comput. Struct.2005;83:1891–1901.

[67] Dinh VN, Du KK, Warnitchai P. Dynamic analysis of three-dimensional bridge-high-speed train interactions using a wheel-rail contact model. Eng Struct. 2009;31:3090–3106.

[68] Antolín P, Zhang N, Goicolea JM, et al. Consideration of nonlinear wheel-rail contact forces for dynamic vehicle-bridge interaction in high-speed railways. J Sound Vib. 2013;332:1231–1251.

[69] Theodosiou, C., Natsiavas, S.: Dynamics of finite element structural models with multiple unilateral constraints. Int. J. Non-Linear Mech. 2009; 44(4), 371-382.

[70] Pfeiffer, F., Glocker, C.: Multibody Dynamics with Unilateral Contacts. Wiley: Singapore;1996

[71] Carter, F.W.: On the action of a locomotive driving wheel. In: Proceedings of the Royal Society of London A: Math-ematical, Physical and Engineering Sciences. London, UK, pp. 151â€“157 (1926)

[72] Zhao X, Li Z. The solution of frictional wheel-rail rolling contact with a 3D transient finite element model: validation and error analysis. Wear. 2011;271:444–452.

[73] Vo KD, Tieu AK, Zhu HT, et al. A 3D dynamic model to investigate wheel–rail contact under

high and low adhesion. Int J Mech Sci. 2014;85:63–75.

[74] Meymand SZ, Keylin A, Ahmadian M. A survey of wheel–rail contact models for rail vehicles.

Veh Syst Dyn. 2016;54:386–428.

[75] Montenegro PA, Calçada R, Pouca NV, et al. Running safety assessment of trains moving over bridges subjected to moderate earthquakes. Earthq Eng Struct Dyn. 2016;45:483–504.

[76] Shen ZY, Hedrick JK, Elkins JA. A comparison of alternative creep force models for rail vehicle dynamic analysis. Veh Syst Dyn. 1983;12:79–83.

[77] ISO. Acoustics-railway applications-measurements of noise emitted by railbound vehicles.

Geneva: ISO 3095; 2013.

[78] Claus H, Schiehlen W. Modeling and simulation of railway bogie structural vibrations. Veh Syst Dyn. 1998;29:538–552.

[79] Xu YL, Zhang N, Xia H. Vibration of coupled train and cable-stayed bridge systems in cross winds. Eng Struct. 2004;26:1389–1406.

[80] Li Y, Qiang S, Liao H, et al. Dynamics of wind-rail vehicle-bridge systems. J Wind Eng Ind Aerodyn. 2005;93:483–507.

[81] Olmos JM, Astiz M. Non-linear vehicle-bridge-wind interaction model for running safety assessment of high-speed trains over a high-pier viaduct. J Sound Vib. 2018;419: 63–89.

[82] Yang, Y.B., Wu, Y.S.. Dynamic stability of trains moving over bridges shaken by earthquakes. J. Sound Vib.2002; 258(1), 65-94.

[83] Cheng, Y.C., Chen, C.H., Hsu, C.T.. Derailment and dynamic analysis of tilting railway vehicles moving over irregular tracks under environment forces. Int. J. Struct. Stab. Dyn. 2017; 17(9), 1750098-1-27.

[84] Nishimura, K., Terumichi, Y., Morimura, T., Sogabe, K.. Development of vehicle dynamics simulation for safety analyses of rail vehicles on excited tracks. J. Comput. Nonlinear Dyn. 2009; 4(1), 011001

[85] Tanabe, M., Wakui, H., Sogabe, M., Matsumoto, N., Tanabe, Y.: An efficient numerical model for dynamic inter-action of high speed train and railway structure including post-derailment during an earthquake. In: 8th International Conference on Structural Dynamics, EURODYN. Leuven, Belgium, pp. 1217-1223 (2011).

[86] Jin, Z.B., Pei, S.L., Li, X.Z., Liu, H.Y., Qiang, S.Z.: Effect of vertical ground motion on earthquake-induced derailment of railway vehicles over simply-supported bridges. J. Sound Vib. 383, 277-294 (2016).

[87] Wu, X.W., Chi, M.R., Gao, H.: Post-derailment dynamic behaviour of a high-speed train under earthquake excita-tions. Eng. Fail. Anal. 2016; 64, 97-110

[88] Weinstock, H.: Wheel climb derailment criteria for evaluation of rail vehicle safety. In: Proceeding of the ASME Winter Annual Meeting, New York, pp. 1-7 (1984.)

[89] Luo, X.. Study on methodology for running safety assessment of trains in seismic design of railway structures. Soil Dyn. Earthq. Eng. 2005; 25(2), 79-91.

[90] Du, X.T., Xu, Y.L., Xia, H.: Dynamic interaction of bridge-train system under non-uniform seismic ground motion. Earthq. Eng. Struct. Dyn. 2012; 41(1), 139-157.

[91] Ju, S.H.. A frictional contact finite element for wheel/rail dynamic simulations. Nonlinear Dyn. 2016; 85(1), 365-374.

[92] Wang SL, Zhai WM. Dynamic responses of high-speed train-track-bridge system under seismic excitations. J Southwest Jiaotong Univ. 2011;46(1):56–62. [in Chinese].

[93] He X, Kawatani M, Hayashikawa T, et al. Numerical analysis on seismic response of Shinkansen bridge-train interaction system under moderate earthquakes. Earthq Eng Eng Vib. 2011;10:85–97.

[94] Zhai WM. Two simple fast integration methods for large-scale dynamic problems in engineering. Int J Numer Methods Eng. 1996;39:4199–4214.

[95] Newmark NM. A method of computation for structural dynamics. J Eng Mech Div. 1959;85:67–94.

[96] Hilber HM, Hughes TJR, Taylor RL. Improved numerical dissipation for time integration algorithms in structural dynamics. Earthq Eng Struct Dyn. 1977;5:283–292.

[97] Wilson EL. Three-dimensional static and dynamic analysis of structures. 3rd ed. Berkeley: Computers and Structures; 2002.

[98] Neves SGM, Azevedo AFM, Calcada R. A direct method for analyzing the vertical vehicle-structure interaction. Eng Struct 2012;34:414-20.

[99] Xia H, Zhang N, Roeck De G. Dynamic analysis of high speed railway bridge under articulated trains. Comput Struct 2003;81(26):2467-78.

[100] Liu K, Reynders E, Roeck De G, Lombaert G. Experimental and numerical analysis of Xia H, a composite bridge for high-speed trains. J Sound Vibrat 2009;320(1):201-20.

[101] Xia H, Zhang N. Dynamic analysis of railway bridge under high-speed trains. Comput Struct 2005;83:1891-901.

[102] Park K, Felippa CA, Gumaste U. A localized version of the method of Lagrange multipliers and its applications. Comput Mech 2000;24(6):476-90.

[103] Park K, Felippa CA, Ohayon R. Partitioned formulation of internal fluid-structure interaction problems by localized Lagrange multipliers. Comput Methods Appl. Mech Eng 2001;190(24):2989-3007.

2 CONSTRAINED DYNAMICAL SYSTEMS – BILATERAL AND UNILATERAL CONSTRAINTS: STATE-OF-THE-ART REVIEW

2.1 State of the art

The study of the dynamics of mechanical systems subjected to bilateral motion constraints is a challenging research topic with a quite long history. Due to its inherent theoretical beauty and the broad range of practical problems that can accommodate, it has attracted the attention of many well-known scientists and engineers, over an extended period. An excellent exposure on the subject, together with a comprehensive historical survey of the pioneering work performed by eminent scientists and natural philosophers like Newton and Leibniz, Lagrange and Euler, Hamilton and Jacobi, can be found in [1]. Since then, a lot of research work has been carried over, and many books and publications have been written on the subject. A partial list of more recent books and papers, referring mostly to mechanical systems in multibody dynamics, control and robotics, includes [2, 3, 4, 5, 6].

In a typical application, the configuration of a mechanical system is fully described by a set of generalized coordinates [7]. When the number of these coordinates is minimal, the system is called free or unconstrained [8, 9]. However, there are many occasions where it is convenient or beneficial to introduce more coordinates than those actually needed for describing the dynamics of a system, composed by either single or multiple bodies. Then, for each extra coordinate, an equation must also be introduced, involving some or all the coordinates and representing a constraint on the motion of the system [10, 11, 12]. Traditionally, these constraints are divided into two distinct categories, leading to entirely different dynamic behavior. The first type, known as holonomic (or geometric or integrable) constraints, impose conditions on the position of the system components. The second type, known as nonholonomic (or kinematic or nonintegrable) constraints, are conditions involving the generalized velocities of the system as well. In both cases, explicit dependence on time means that the constraint is rheonomic. Otherwise, the constraint is called scleronomic [6, 7].

Among the many theoretically difficult and open issues in this scientific area, a quite important research topic is related to the derivation of a new set of equations of motion, when an additional set of constraints is imposed on the motion of a mechanical system. Several theoretical or more numerically oriented studies have already been reported on the subject, including [13, 14, 15, 16] and [17, 18, 19, 20, 21, 22], respectively. However, there remain a plethora of topics deserving of further investigation. Precisely, in paper [23] a new solid theoretical foundation for producing the equations of motion of constrained systems in a systematic, consistent and precise way is derived. The mathematical structure of this problem makes natural the application of some fundamental concepts of differential geometry. Primarily, the motion of a mechanical system of any complexity is represented as the motion of a single point on a path of a manifold. Based on the form of the motion constraints, this manifold is in general non-Euclidean (non-flat), with curvature and torsion, leading to significant consequences on its geometrical properties [4, 24, 25].

More specifically, the emphasis of the work developed in [23] is placed on mechanical systems subjected to scleronomic constraints, and the attention is focused on deriving an appropriate set of conditions so that the form of Newton's law remains invariant on the new manifold. The main idea is to apply tools and concepts of general (non-Riemannian) geometries, which are shown to be more natural and beneficial in studying the dynamics of mechanical systems, especially in the presence of large rigid body rotation. In particular, carrying out the analysis, this is found to lead to two conditions. The first condition specifies completely the metric and the second condition restricts the form of the connection of the new manifold. Then, it is shown that the condition on the connection presents some similarities but is, in fact, weaker than a related condition imposed frequently in the literature. The latter condition relies on a metric compatibility requirement for the connection of Riemannian manifolds; it is purely geometric and specifies the connection on the new manifold [24, 26, 27].

On the other hand, the condition imposed on the connection by the present work is based on the dynamics and provides some freedom in choosing its components. It is shown to be equivalent to the conservation of mass of the system. This freedom is proved to be valuable in practical applications, like in systems involving large rigid body rotation, where it is used to preserve certain group properties of the configuration space [28].

Moreover, employing this new condition on the connection helps in clarifying some issues related to the correct form of the equations of motion of constrained systems. This condition includes the specification of the requirements so that Newton's law can be applied on the tangent (and not on the dual) space of a manifold as well as an illustration of the difference between anholonomic and vakonomic dynamics [3, 4, 27]. It is important to note that the entire study is performed within the simple framework of Newtonian Mechanics, involving no Lagrangian or Hamiltonian formalism [4, 24]. Besides the theoretical contributions mentioned, the results of this study will provide a solid foundation for developing appropriate and efficient numerical schemes for mechanical systems subjected to motion constraints.

Despite all the efforts, there remain a plethora of important and challenging theoretical aspects on the subject that remain open in the case of unilateral constraints. Consequently, new studies in this area are of large practical significance, since their successful execution can cause a considerable impact in formulating and solving complicated engineering problems in a better and more efficient way. In this area, the category of constraints examined involve contact, with or without friction, between the interacting bodies and generate forces, which can act in one direction only, to avoid interpenetration. Such constraints are described by inequalities and are known as unilateral.

There exists a vast literature about unilateral constraints. Based on the type of analytical tools employed, the available approaches can roughly be divided into two main groups. In the first group, the research is conducted by applying standard analytical tools [29-35], while the work presented in the second group is heavily based on special concepts of non-smooth mechanics, involving advanced material from convex analysis [36-44]. These methodologies apply the classical laws of motion up to a specific time where contact is established between the interacting bodies. Then, based on the relatively short duration of the contact phase, it is assumed that the position of the system components does not change appreciably during the contact phase, while the forces developed become excessively large (impulsive), causing a significant change in the velocities of the impacting bodies. This brings the need to introduce an approximate impact law, predicting the sudden change in the velocities during contact. Typically, such a change is predicted by using Newton's or Poisson's restitution coefficients or other similar concepts [30, 35].

In most of the previous studies on the subject, the approach leading to a prediction of the post-impact velocity is algebraic. However, in some cases, the time interval where the contact event takes place is considered to be short but finite and the contact process is modeled by an approximate set of ordinary differential equations (ODEs), using the corresponding normal impulse as the independent variable [7, 9-11, 13]. This type of analysis is known as a Darboux-Keller approach and is currently confined to contact between two particles or rigid bodies [40].

The work presented in [45] can be considered as the first step towards a systematic extension and generalization of this type of analysis to systems possessing general dynamic properties, by adopting the spirit and framework of Analytical Mechanics. More specifically, the main objective of this work was to present a new formulation for an important class of mechanical systems, involving unilateral constraints. Building upon the strong and intimate relation between mechanics and differential geometry [3, 6, 23, 46-47], the methodology developed is based and continues along the lines of some recent work of the authors on mechanical systems subject to bilateral constraints [24, 48], together with some new and sound geometrical concepts. Namely, some powerful tools of b-geometry are borrowed and utilized for setting up a suitable theoretical framework, in order to help the study of mechanical systems with unilateral constraints [47, 49-51]. In particular, paper [45] focuses on the investigation of systems involving a single frictionless contact. In this way, several new mathematical ideas are introduced and explained in a problem characterized by a sufficiently high but not excessive level of mechanical complexity. Moreover, the analysis is performed in a way providing a solid foundation for a subsequent extension and application of similar methodologies to more complex mechanical problems, involving friction or even multiple contact events [36-44].

The work in paper [45] takes a different perspective and route than previous studies on the subject. The main contribution is that Newton's second law of motion is applied in full form even during the short time interval where the contact event occurs. Therefore, there is no need to define other concepts, like the restitution coefficient. However, the most fundamental aspect of this work is that the analysis performed remains entirely smooth. Namely, both the generalized velocities and the corresponding momenta remain smooth and bounded during the entire contact phase. This is obtained by using some remarkable tools of b-geometry, after relating properly the theory of manifolds with a boundary to the class of systems examined [49-51]. This relation is established by using the equation in the unilateral constraint to define a boundary hypersurface within the original configuration space. This step is followed by the introduction of an appropriate vector bundle over the configuration manifold, consisting of smooth velocity fields only. This, in turn, provides a solid foundation for evaluating the covariant derivative appearing in the corresponding law of motion over the manifold [3,25,47,48]. As usual, this derivative requires knowledge of a suitable metric and a connection on the configuration manifold. The most striking characteristic of this theory is the rapid change in the metric and the connection components of the configuration manifold within a thin layer starting on the boundary hypersurface, defined by the unilateral constraint. Specifically, the metric component along the normal to the boundary increases rapidly as the figurative particle, representing the motion of the mechanical system within the constrained manifold, approaches the boundary, so that as the component of the generalized velocity in the normal to the boundary tends to zero the kinetic energy of the system remains bounded. Apart from more accurate and realistic modeling of the impact phase, the results of the proposed analysis lead to the development of more efficient and robust numerical schemes for predicting the response and developing better control algorithms for mechanical systems involving contact.

Also in paper [52], that is a continuation of earlier work of the authors [45, 58] and applies to a particular class of constrained mechanical systems, involving a single contact event with friction. The approach taken is novel compared to previous studies on the subject. In particular, the analysis applied is carried out within the classical framework of analytical dynamics and can handle systems with general properties, including rigid and deformable bodies, in an equally efficient manner. The formulation is based on a proper application of Newton's law of motion during the contact phase. The outcome is an entirely continuous formulation, in contrast to the approaches based on non-smooth techniques. This is achieved through the use of some key ideas and concepts of b-geometry, referring to manifolds with a boundary, which provides a natural and robust setting for studying mechanical systems subject to a unilateral constraint [49, 51]. After defining the boundary of the configuration manifold and determining the essential geometric properties needed for the application of the law of motion inside a thin layer starting at this boundary, it is shown that the dominant dynamics during a single frictional collision is described by a set of three coupled ODEs. These equations describe action in a three-dimensional distribution of the configuration manifold, which is related directly to the action in the physical space, where the contact event examined takes place. Using time as an independent variable presents an advantage over the Darboux-Keller approaches since it provides a valuable time scale for investigating the part of the motion inside the boundary layer. However, the most striking feature is that the general spirit of the new approach is entirely different. Instead of modeling the contact with stiff springs, which demands to give up the rigidity assumption in case of a rigid body contact, application of the theory for a manifold with boundary reveals that the associated contact action is modeled by a massive change in the inertia properties [49], in combination with the appearance of a strong repulsive force. More specifically, the components of the metric and the connection employed along the normal to its boundary vary in a quite rapid fashion in the vicinity of the boundary in the configuration space. This causes a fast deceleration of the figurative particle modeling the motion of the mechanical system in the configuration manifold, as it approaches the boundary. Also, once this particle enters the boundary layer, it is pushed away from it by a strong repulsive force, which is exerted on it until its exit from the boundary [48].

Previous studies on the subject have demonstrated that the presence of friction is related to the occurrence of a large variety of phenomena during contact [35, 40]. Particular emphasis is put on modeling the frictional effects in an accurate way as well as on establishing a proper mapping between the significant kinematic and kinetic quantities in the configuration space and the physical space. The formulation is applicable for a general friction law.

Apart from several notable exceptions [40, 53, 54], the vast majority of previous efforts have focused on mechanical systems subject to either bilateral (i.e., equality) or unilateral (i.e., inequality) constraints alone. The paper [55] is a continuation of previous studies of the authors on constrained systems [23, 45, 48, 52]. In particular, systems involving bilateral constraints were first examined in [48] and [23]. Then, systems involving a single unilateral constraint in the absence and presence of friction effects were investigated in [45] and [52], respectively. In both cases, a minimal set of generalized coordinates was selected for convenience in introducing some new useful concepts, originating from differential geometry [6, 25, 49, 56]. Therefore paper [55] is an extension of them to systems involving a set of redundant coordinates, which are associated with an addition of some bilateral constraints.

The presented formulation in paper [55] is developed within an Analytical Dynamics framework and employs some fundamental results from the theory of manifolds with boundary [49, 56]. This boundary defines a subspace of the original configuration manifold where the motion of the figurative particle, representing the motion of the system examined, is allowed to take place. Also, when the system is subject to a set of bilateral constraints as well, each of these constraints causes a specific correction to the impact direction. Therefore, a complete set of geometric properties is established, providing the ground for a consistent application of Newton's law of motion through the entire impact phase. This generates a complete and elegant picture of the dynamics and provides a useful insight into the phenomena taking place over the short time interval of an impact. For instance, the essential dynamics is described by a single ODE, representing action in the vicinity of the boundary and along a direction determined entirely by the simultaneous application of the unilateral and bilateral constraints. Namely, as the fictitious particle approaches the boundary along that direction, its inertia increases rapidly, causing its fast deceleration. At the same time, a strong repulsive force is generated, pushing this particle away from the boundary. On the other hand, when the friction effects are essential, the dominant dynamic action is found to take place in at most three-dimensional subspace, generated by bringing the friction cone from the physical to the configuration space. In both the frictionless and frictional cases, the equations describing the impulses developed due to the bilateral constraints decouple from the equations describing the kinematics of the system, when a suitable coordinate system is selected. This explains why systems subject to bilateral constraints were possible to be analyzed before by the same methods employed for systems of free bodies [8,10]. Also, it is demonstrated that the classical Darboux-Keller approach is a special case of the proposed formulation, obtained when the boundary force is sufficiently large, and the pre-impact speed is low so that the figurative particle does not have a chance to virtually enter into the boundary layer.

References

[1] Lanczos, C.: The Variational Principles of Mechanics. University of Toronto Press, Toronto, Canada (1952).

[2] Bauchau, O.A.: Flexible Multibody Dynamics. Springer Science+Business Media B.V., London (2011).

[3] Bloch, A.M.: Nonholonomic Mechanics and Control. Springer-Verlag New York Inc., New York (2003).

[4] Marsden, J.E., Ratiu, T.S.: Introduction to Mechanics and Symmetry, second ed. Springer Science+Business Media Inc., New York (1999).

[5] Murray, R.M., Li, Z., Sastry, S.S.: A Mathematical Introduction to Robot Manipulation. CRC Press, Boca Raton, Florida (1994).

[6] Papastavridis, J.G.: Tensor Calculus and Analytical Dynamics. CRC Press, Boca Raton (1999).

[7]Greenwood, D.T.: Principles of Dynamics. Prentice-Hall Inc., Englewood Cliffs, New Jersey (1988).

[8] Goudas, I., Stavrakis, I., Natsiavas, S.: Dynamics of slider-crank mechanisms with flexible supports and non-ideal forcing. Nonlinear Dyn. 35, 205-227 (2004).

[9] Nayfeh, A.H., Balachandran, B.: Applied Nonlinear Dynamics. Wiley-Interscience, New York (1995).

[10] Geradin, M., Cardona, A.: Flexible Multibody Dynamics. John Wiley & Sons, New York (2001).

[11] Nikravesh, P.E.: Computer-Aided Analysis of Mechanical Systems. Prentice-Hall Inc, Englewood Cliffs, New Jersey (1988).

[12] Shabana, A.A.: Dynamics of Multibody Systems, third ed. Cambridge University Press, New York (2005).

[13] Essen, H.: On the geometry of nonholonomic dynamics. ASME J. Appl. Mech. 61, 689-694 (1994).

[14] Kane, T.R., Levinson, D.A.: Dynamics: Theory and Applications. McGraw-Hill, New York (1985).

[15] Muller, A.: Motion equations in redundant coordinates with application to inverse dynamics of constrained mechanical systems. Nonlinear Dyn. 67, 2527-2541 (2012).

[16] Udwadia, F.E., Kalaba, R.E., Phohomsiri, P.: Mechanical systems with nonideal constraints: explicit equations without the use of generalized inverses. ASME J. Appl. Mech. 71, 615-621 (2004).

[17] Bayo, E., Ledesma, R.: Augmented Lagrangian and mass-orthogonal projection methods for constrained multibody dynamics. Nonlinear Dyn. 9, 113-130 (1996).

[18] Brüls, O., Cardona, A., Arnold, M.: Lie group generalized-α time integration of constrained flexible multibody systems. Mechanism and Machine Theory 48, 121-137 (2012).

[19] Crouch, P., Grossman, R.: Numerical integration of ordinary differential equations on manifolds. J. Nonlinear Science 3, 1–33 (1993).

[20] Kurdila, A.J., Junkins, J.L., Hsu, S.: Lyapunov stable penalty methods for imposing holonomic constraints in multibody system dynamics. Nonlinear Dyn. 4, 51-82 (1993).

[21] Munthe-Kaas, H.: Runge–Kutta methods on Lie groups. BIT 38, 92–111 (1998).

[22] Negrut, D., Haug, E.J., German, H.C.: An implicit Runge-Kutta method for the integration of differential-algebraic equations of multibody dynamics. Multibody System Dyn. 9, 121-142 (2003).

[23] S. Natsiavas, E. Paraskevopoulos, A set of ordinary differential equations of motion for constrained mechanical systems, Nonlinear Dyn. 79 (2015), 1911-1938.

[24] Arnold, V.I.: Mathematical Methods of Classical Mechanics, second ed. Springer-Verlag, Berlin (1989).

[25] Frankel, T.: The Geometry of Physics: An Introduction. Cambridge University Press, New York (1997).

[26] Shabanov, S.V.: Constrained systems and analytical mechanics in spaces with torsion. J. Phys. A: Mathematical and General 31, 5177-5190 (1998).

[27] Guo, Y.-X., Wang, Y., Chee, G.Y., Mei, F.-X.: Nonholonomic versus vakonomic dynamics on a Riemann-Cartan manifold. J. Math. Phys. 46, 0692902 (2005).

[28] Paraskevopoulos, E., Natsiavas, S.: A new look into the kinematics and dynamics of finite rigid body rotations using Lie group theory. Int. J. Solids Struct. 50, 57-72 (2013).

[29] J.B. Keller, Impact with friction, ASME J. Appl. Mech. 53 (1986) 1–4.

[30] R.M. Brach, Mechanical Impact Dynamics, John Wiley, New York, 1991.

[31] J.A. Batlle, On Newton's and Poisson's rules of percussive dynamics, ASME J. Appl. Mech. 60 (1993) 376-381.

[32] W.J. Stronge, Swerve during three-dimensional impact of rough rigid bodies, ASME J. Appl. Mech. 61 (1994) 605–611.

[33] V. Bhatt, J. Koechling, Three-dimensional frictional rigid-body impact, ASME J. Appl. Mech. 62 (1995) 893-898.

[34] V.I. Babitsky, Theory of Vibro-Impact Systems and Applications, Springer-Verlag, Berlin, 1998.

[35] W.J. Stronge, Impact Mechanics, Cambridge University Press, Cambridge, UK, 2000.

[36] J.J. Moreau, P.D. Panagiotopoulos (eds.), Nonsmooth Mechanics and Applications. CISM Courses and Lectures, Vol. 302, Springer-Verlag, Wien, 1988.

[37] M. Fremond, Rigid bodies collisions, Phys. Lett. A 204 (1995) 33-41.

[38] F. Pfeiffer, C. Glocker, Multibody Dynamics with Unilateral Contacts, J. Wiley & Sons, New York, 1996.

[39] M. Jean, The non-smooth contact dynamics method, Comput. Methods Appl. Mech. Eng. 177 (1999) 235-257.

[40] B. Brogliato, Nonsmooth Mechanics: Models, Dynamics and Control (3rd ed.), Springer-Verlag, London, 2016.

[41] C. Glocker, Set-Valued Force Laws, Dynamics of Non-Smooth Systems, Springer, Berlin, 2001.

[42] C. Glocker, An Introduction to Impacts. In: Nonsmooth Mechanics of Solids. CISM Courses and Lectures Vol. 485 (eds. Haslinger J., Stavroulakis G.), pp. 45–102, Springer Verlag, Wien, New York, 2006.

[43] R.I. Leine, H. Nijmeijer, Dynamics and Bifurcations of Non-Smooth Mechanical Systems, Springer-Verlag, Berlin, 2004.

[44] C. Theodosiou, S. Natsiavas, Dynamics of finite element structural models with multiple unilateral constraints, Int. J. Non-Linear Mech. 44 (2009) 371-382.

[45] Paraskevopoulos, E. and Natsiavas, S., 2017, "A Geometric Solution to the General Single Contact Frictionless Problem by Combining Concepts of Analytical Dynamics and b-Calculus," Int. J. Non-Linear Mech., 95, pp. 117-131.

[46] R. Abraham, J.E. Marsden, T.S. Ratiu, Manifolds, Tensor Analysis and Applications, Applied Mathematical Sciences, Vol. 75, Springer-Verlag, New York, 1988.

[47] S. Natsiavas, E. Paraskevopoulos, Application of Newton's law of motion to constrained mechanical systems possessing configuration manifolds with time dependent geometric properties, Nonlinear Dyn. 85 (2016) 2583-2610.

[48] E. Paraskevopoulos, S. Natsiavas, On application of Newton's law to mechanical systems with motion constraints, Nonlinear Dyn. 72 (2013) 455-475.

[49] R.B. Melrose, The Atiyah-Patodi-Singer index theorem, Research Notes in Mathematics, Vol. 4, A.K. Peters Ltd., Wellesley, MA, 1993.

[50] M. Lesch, H. Moscovici, M.J. Pflaum, Connes–Chern character for manifolds with boundary and eta cochains, 2000 Mathematics Subject Classification. Primary 58Jxx, 46L80; Secondary 58B34, 46L87.

[51] D. Krupka, D. Saunders (eds), Handbook of Global Analysis, Elsevier, Amsterdam, Netherlands, 2008.

[52] Natsiavas, S., and Paraskevopoulos, E., 2018, "An Analytical Dynamics Approach for Mechanical Systems Involving a Single Frictional Contact Using b-Geometry," Int. J. Solids Struct., accepted (DOI: 10.1016/ j.ijsolstr.2018.05.008).

[53]Aghili, F., 2011, "Control of Redundant Mechanical Systems Under Equality and Inequality Constraints on Both Input and Constraint Forces," ASME J. Comput. Nonlinear Dyn., 6, 031013.

[54]Brogliato, B., 2014, "Kinetic Quasi-Velocities in Unilaterally Constrained Lagrangian Mechanics With Impacts and Friction," Multibody Syst. Dyn., 32, 175–216.

[55] S Natsiavas, E Paraskevopoulos, A Boundary Layer Approach to Multibody Systems Involving Single Frictional Impacts, J. Comput. Nonlinear Dynam. Jan 2019, 14(1): 011002

[56] Tu, L. W., 2011, *An Introduction to Manifolds* (2nd ed.), Springer Science+Business Media, New York.